The Core

General Equilibrium (without Production) or Exchange (Chapter 31)



- Events in one market have effects on other markets (spillovers)
- Demand for x depends upon prices of complements, substitutes; income
- Supply of x depends upon factor prices
- Previously, we've taken these as given- doing *partial* equilibrium analysis
- But its important to understand interdependence of marketsgeneral equilibrium analysis

Partial equilibrium analysis says that competitive markets yield efficient outcomes—is this still true in general equilibrium?

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General Equilibrium

Our approach:

- Simple environment—the entire economy
 - 2 kinds of goods
 - 2 people
- Focus on exchange
 - Abstract away from production of new goods
 - Give people endowments
 - Specify preferences
 - Allow them to trade
- Make predictions about behavior of utility-maximizers
- Evaluate welfare

General vs. Partial Equilibrium 00●0	The Edgeworth Box	The Contract Curve	The Core
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Endowment Economy

- Consumers A and B; goods 1 and 2
- Endowments: $\omega^A = (\omega_1^A, \omega_2^A)$ and $\omega^B = (\omega_1^B, \omega_2^B)$
- Example: $\omega^A = (6, 4)$ and $\omega^B = (2, 2)$
- This means total endowment of good 1 is $\omega_1^A + \omega_1^B = 6 + 2 = 8$ and of good 2 is $\omega_2^A + \omega_2^B = 4 + 2 = 6$

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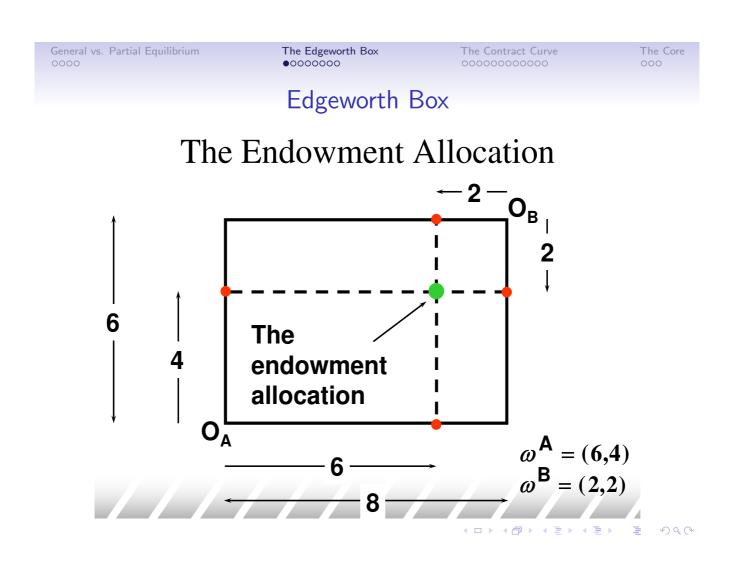
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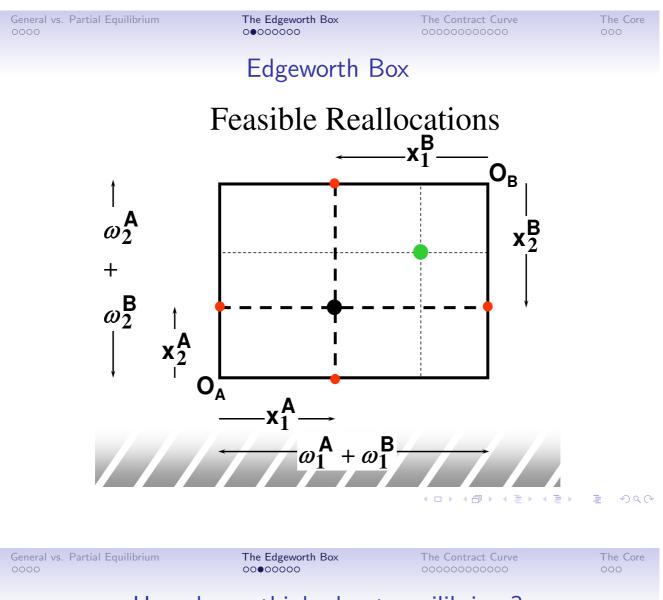
Allocations

- Endowment represents where people start, but through trade, their allocations may change
- General allocation or consumption: $x^A = (x_1^A, x_2^A)$ and $x^B = (x_1^B, x_2^B)$
- (x^A, x^B) is *feasible* if it uses at most the aggregate endowment:

 $x_1^A + x_1^B \leq \omega_1^A + \omega_1^B \text{ and } x_2^A + x_2^B \leq \omega_2^A + \omega_2^B$

- Helpful graphical tool: Edgeworth Box
- Allows us to simply depict all feasible allocations





How do we think about equilibrium?

- In partial equilibrium analysis:
 - Treat each good separately
 - Find p and q that equate supply and demand
- But this is general equilibrium analysis: where do supply and demand come from?
- A and B can trade with each other
- For everything to be balanced, the amount that A gives up has to equal amount that B receives (for each good, and vice versa)
- In other words *Supply* = *Demand* for each good
- This will determine prices for each good
- How do we find supply and demand curves?
- Go back to utility maximization problem
- Need to specify preferences to do this

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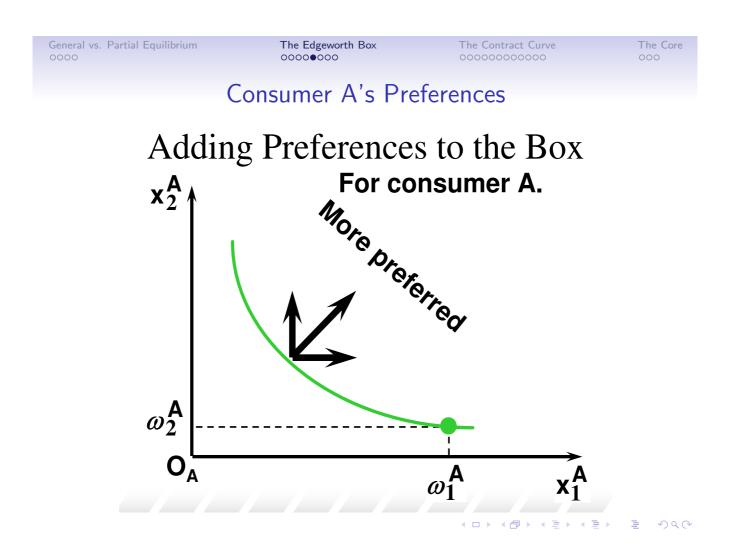
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Utility maximization

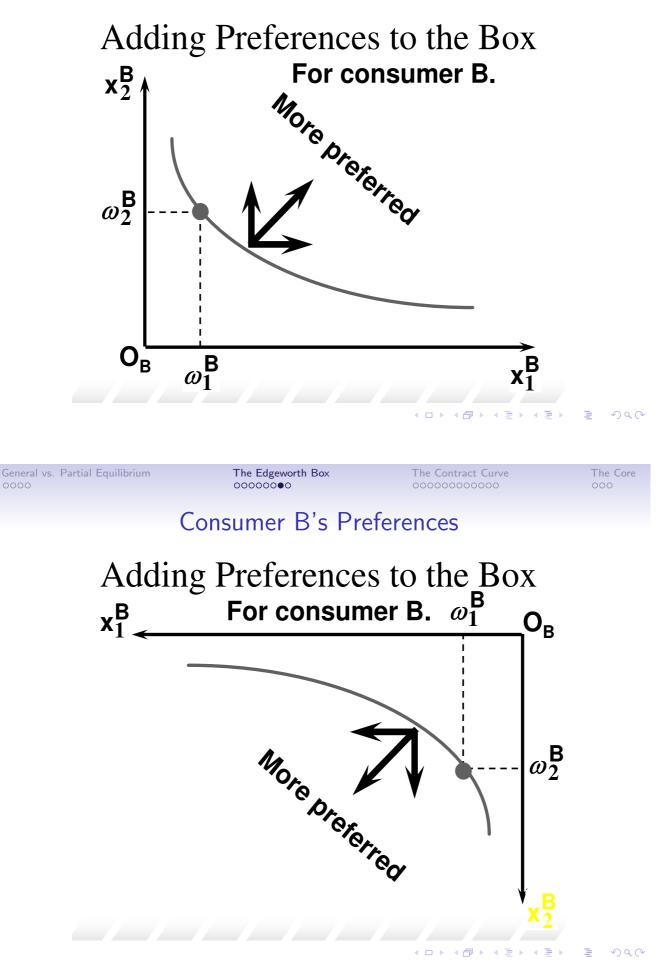
- Preferences are given
- Given prices for each good, endowment bundle serves as income
- Can write down budget constraint

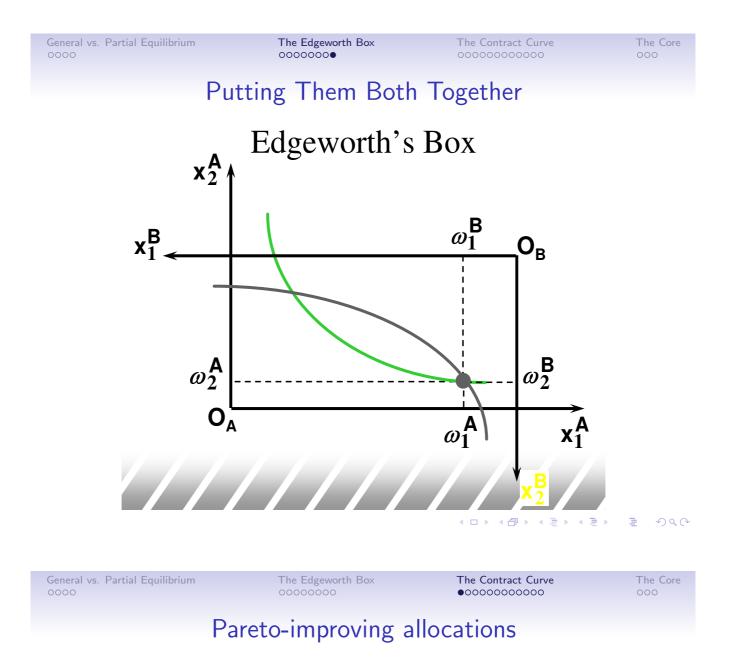
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p_1x_1+p_2x_2\leq p_1\omega_1+p_2\omega_2
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- Solve utility maximization problem
- Gives you optimal allocation, as a function of price ratio
- $x_1^* \omega_1 > 0$ means person demands more of good 1
- $x_1^* \omega_1 < 0$ means person is willing to supply good 1
- Key question: what prices will make it so that A demand exactly as much of each good as B supplies?

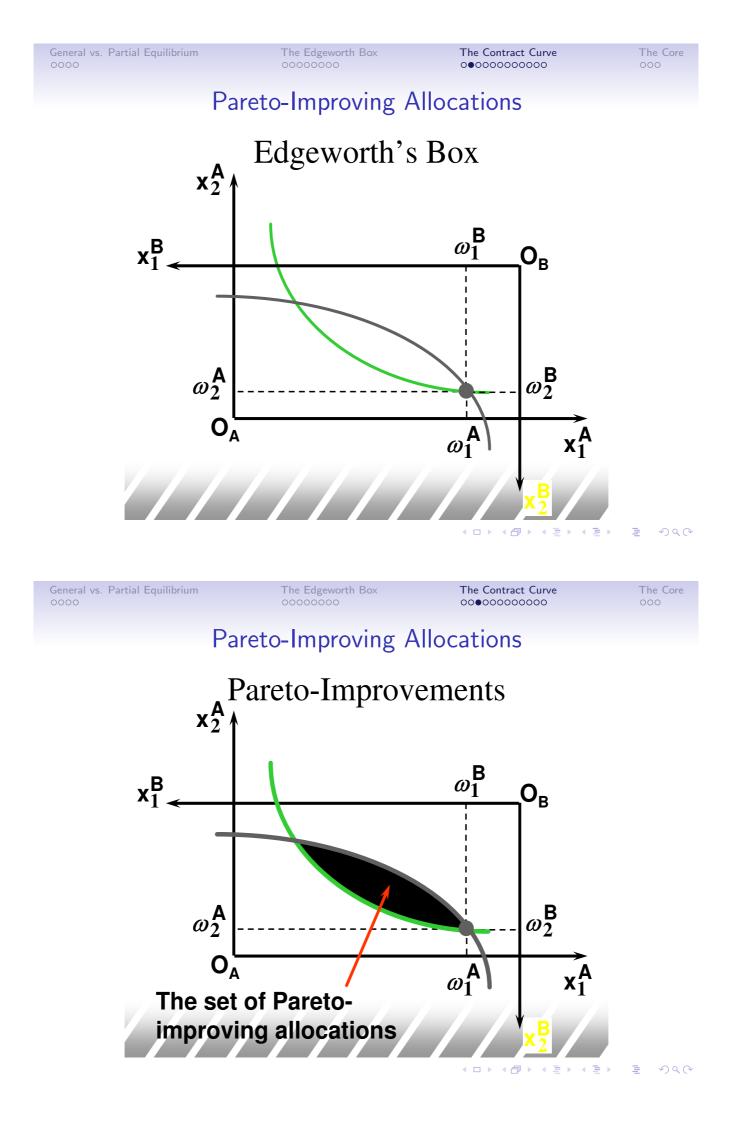


Consumer B's Preferences



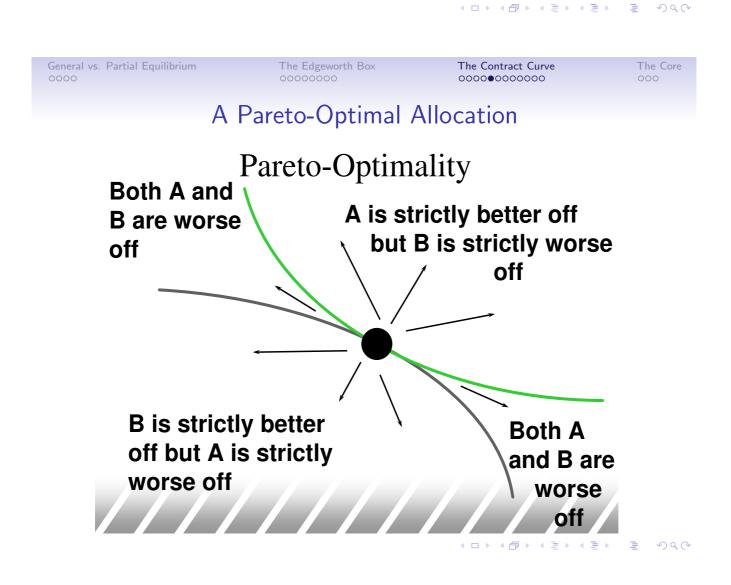


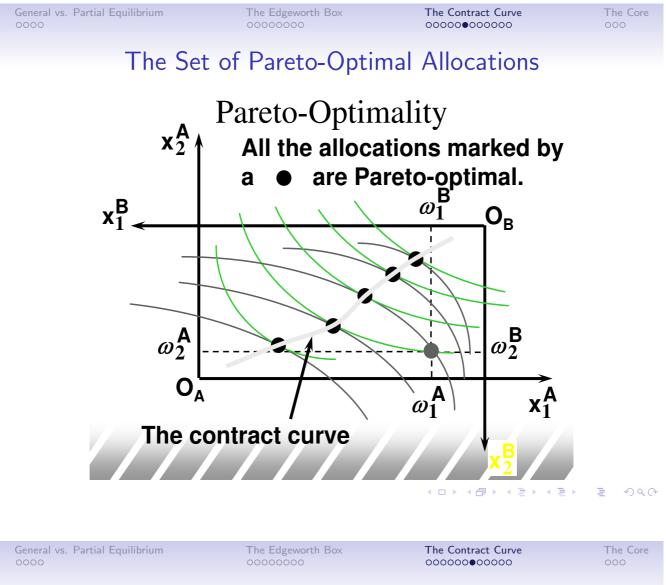
- Given a particular allocation, a *Pareto-improving* allocation improves the welfare of at least one consumer *without reducing the welfare of another*.
- How do we depict Pareto-improving allocations in the Edgeworth box?



Pareto-optimal allocations

- An allocation is *Pareto-optimal* if it is *feasible* and there is other feasible allocation that is a Pareto-improvement over it.
- In other words, there is no was to make anyone better off without making someone worse off.
- The set of all Pareto-optimal allocations is called the contract curve.





Pareto-optimal Allocations

- From the figures, we can see that an allocation at which the indifference curves of the two consumers are tangent must be Pareto-optimal
- Tangency implies they have the same slope
- What is the slope of an indifference curve? The Marginal rate of substitution (MRS)!
- Condition for Pareto-optimality:

$$MRS^{A} = \frac{\frac{\partial u^{A}(x_{1}^{A}, x_{2}^{A})}{\partial x_{1}^{A}}}{\frac{\partial u^{A}(x_{1}^{A}, x_{2}^{A})}{\partial x_{2}^{A}}} = \frac{\frac{\partial u^{B}(x_{1}^{B}, x_{2}^{B})}{\partial x_{1}^{B}}}{\frac{\partial u^{B}(x_{1}^{B}, x_{2}^{B})}{\partial x_{2}^{B}}} = MRS^{B}$$

• We also require feasibility:

$$x_1^A + x_1^B = \omega_1^A + \omega_1^B$$
 and $x_2^A + x_2^B = \omega_2^A + \omega_2^B$

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Example

Identifying Pareto-optimal allocations

- Recall total endoments: $\omega_1^A + \omega_1^B = 6 + 2 = 8$ and $\omega_2^A + \omega_2^B = 4 + 2 = 6$ • Let $u^A(x_1^A, x_2^A) = \ln(x_1^A) + 2\ln(x_2^A)$ and
- $u^{B}(x_{1}^{B}, x_{2}^{B}) = \ln(x_{1}^{B}) + 2\ln(x_{2}^{B}).$
- MRS of consumer A:

$$MRS^{A} = \frac{\frac{1}{x_{1}^{A}}}{\frac{2}{x_{2}^{A}}} = \frac{x_{2}^{A}}{2x_{1}^{A}}$$

Similarly,

$$MRS^{B} = \frac{\frac{1}{x_{1}^{B}}}{\frac{2}{x_{2}^{B}}} = \frac{x_{2}^{B}}{2x_{1}^{B}}$$

• So a Pareto-optimum is a feasible allocation for which

$$\frac{x_2^A}{2x_1^A} = \frac{x_2^B}{2x_1^B}$$

General vs. Partial Equilibrium 0000	The Edgeworth Box	The Contract Curve	The Core

Example

Finding all Pareto-optimal allocations (deriving the contract curve)

• We can simplify tangency condition to:

$$\frac{x_2^A}{x_1^A} = \frac{x_2^B}{x_1^B}$$

• Recall endowment/feasibility requirement:

$$x_1^A + x_1^B = 8$$
 and $x_2^A + x_2^B = 6$

• Re-write tangency condition, substituting $x_1^B = 8 - x_1^A$ and $x_2^B = 6 - x_2^A$:

$$\frac{x_2^A}{x_1^A} = \frac{6 - x_2^A}{8 - x_1^A}$$

or

$$x_2^{\mathcal{A}} = \frac{3}{4}x_1^{\mathcal{A}}$$

- This is the equation of the contract curve.
- In this case, it's just the diagonal of the rectangle